Individual Rationality in Collective Choice

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Abstract

This paper studies the rationality of an individual player in sequential games of perfect information played with other players who are not necessarily rational. The paper proposes a set of properties on the choice behavior and shows that they are equivalent to the rationality of an individual player at the initial node independently of the behavioral norm of the other players. Furthermore, I show that the choice of subgame perfect equilibrium paths is obtained as a special case where all players involved in the environment are individually rational. Therefore, the paper offers a testable condition both for individual rationality and collective rationality in sequential games.

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Keywords: Rationalizability, Revealed preference, Subgame perfect equilibrium.

1 Introduction

A game is a description of strategic interactions among players. The players involved in a game simultaneously or dynamically choose their actions, and their payoffs are determined by a profile of chosen actions. A number of solution concepts for a game, such as the Nash equilibrium or the subgame perfect equilibrium, have been developed in the literature in order to study collective choice of actions by the players. In turn, these solution concepts are widely applied in economic analysis to provide a credible prediction for the choice of economic agents who live in a situation approximated by the game.

However, the vast majority of solution concepts for games are defined by preference relations (usually represented by payoff functions) of the players, while in practice only choice behavior of actions are observable from data. So, even when a certain solution concept appears reasonable to make predictions of the outcome of a game, we may not be able to apply such a concept unless the players' preference relations are known to the outside observer and the players are indeed rational decision makers. Hence, it is of importance for the empirical contents of the game theory that there is a method that allows us to, based on observable data set, test the rationality of players involved in a collective choice environment and reveal their preference relations.

This paper poses the following question: When we observe the choice of actions made by players involved in a collective choice environment, but not their preference relations, how can we test whether the players rationally choose their actions according to their preference relations? This question parallels *the revealed preference theory*

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	Individual rationality	Collective rationality
Individual choice data	Samuelson (1938) Houthakker (1950) Arrow (1959) Sen (1971)	n/a
Collective choice data	This paper Gul and Pesendorfer (2005)	Sprumont (2000) Ray and Zhou (2001) Carvajal et al. (2010) Chiappori et al. (2012) Bossert and Sprumont (2013)

Table 1: Related literature

due to Samuelson [11], followed by Arrow [1], Houthakker [7], Sen [12], and many others. In this literature, the authors assume observability of individual choice behavior from various choice sets, and they seek the testable properties on the choice data under which the observed choice can be represented by maximization of a preference relation. Furthermore, provided that the choice data satisfies such a rationalizability condition, the theory implements a method of revealing a preference relation for the individual decision maker, thereby offering the theoretical foundation for the applications of the consumer theory which typically takes a consumer's preference relation as a primitive.

This paper, on the other hand, departs from the classical revealed preference theory by assuming the observability of *collective* choice behavior from a variety of collective choice problems and by studying the rationalizability of decision makers (or players) involved in such an choice environment. As a matter of fact, reflecting the significant interest for the rational choice models in multi-player strategic environments, considerable effort has been made in the literature to provide testable properties of rational decision makers in this framework. For example, Sprumont [13] takes the games of simultaneous moves and investigates axiomatization of Pareto optimality and Nash equilibrium. On the other hand, Ray and Zhou [10] assume observability of paths chosen by the players in extensive games and characterize the subgame perfect equilibrium. In other contexts, Carvajal et al. [4] develop the revealed preference theory for Cournot competition games, and Chiappori et al. [5] examine the testability of Nash solution in two-player bargaining games. Furthermore, Bossert and Sprumont [3] recently show that, if only terminal nodes (i.e. outcomes) achieved by the players are observable, but not the structure of games, number of players, nor their preferences, then every choice function is rationalizable as a result of backwards induction of some game.

It is, however, worth noting that the existing literature has focused on the *collective* rationality in collective choice environments, and little attention has been paid to the *individual* rationality in these frameworks. After all, rationality is a concept first formed for individuals, and in a collective choice problem, it is a necessary requirement for the collective rationality. In addition, recent findings in behavioral economics and in the literature of bounded rationality suggest that decision makers in reality often violate the prediction of the rational choice theory. Therefore, even if we find it difficult to fully support the collective rationality against observed choice data, it is still of interest to identify a set of players, if not all, who behave rationally. A question thus leads to finding a testable characterization of the individual rationality in the collective choice environment. This is what this paper is after. (See Table 1.)

In this regard, this paper also relates to the work by Gul and Pesendorfer [6], where they study intertemporal decision problems of a single decision maker with changing tastes. In case a decision maker faces time inconsistency in preferences, the model of consistent planning, originally proposed by Strotz [14], views the decision maker at different points in time as different players and solves for an action path that she can actually follow through. Gul and Pesendorfer take as the primitive a preference relation of the decision maker at the initial period over such decision problems (that is, games with multi-selves) and characterize the condition under which we can view as if this preference relation arises from Strotz model. Notably, motivated by behavioral decision making under temptation, they axiomatize the model where multi-selves are only boundedly rational, which they call *weak Strotz model*. To clarify the objective of their work from this, however, it should be remarked that the purpose of their model is to offer a testable property for collective bounded rationality. The weak Strotz model assumes a certain behavioral norm for all players, which for example, reduces to the full rationality in case the intertemporal decision problem consists of only two periods.[1]

This paper aims to characterize the individual rationality in the collective choice environment, without relying on any behavioral assumption on the other players. To be more specific, I take a plain setup of sequential games where linearly ordered players make their actions one after another, and observability is assumed on paths followed by the players in such games. I focus on a person, called player 1, who chooses an action first among all the players (i.e. standing at the initial node) and search a testable axiomatization of the individual rationality for this player. What defines the individual rationality in this context is a remark that, if player 1 is in fact rational, she should form a history-dependent belief of actions chosen by the subsequent players and choose her own action in order to achieve an optimal path among those that are actually followed by the players. It is here important to note that the other players may make *unrationalizable* actions, but even if so, player 1 takes such into consideration and choose her action to achieve the best outcome among those that she can achieve. The main theorem of the paper provides an axiomatization on the observed choice data that is equivalent to the rationality of player 1 in this sense.

The collective rationality is reconstructed from the individual rationality. In fact, I show that the set of observed paths chosen by the players in a sequential game coincides with a set of subgame perfect equilibrium paths, provided that all players in the game is individually rational. But the individual rationality of every involved player is testable through a plain adjustment of the aforementioned result. Thus, this paper also provides an axiomatic characterization of the subgame perfect equilibrium in extensive games of perfect information parallel to the work of Ray and Zhou [10].[2]

The paper is structured as follows. First, Section 2 places all preliminary concepts used throughout the paper including a formulation of sequential games amenable to the

[1] To be precise, the weak Strotz model assumes that the decision maker at any given period has a preference relation over continuation games. For the rational player, the preference over continuation games is induced from her preference over ultimate outcomes of the game that she can actually achieve. The weak Strotz model requires this only at the initial period but not those following. The decision maker at the subsequence period may be myopic and prefer one continuation game over another even if the former leads to an inferior outcome in the end. Note that, however, if there are only two periods in the intertemporal decision problem, then the continuation games at the second period are just terminal nodes. Therefore, the bounded rationality argued above trivially coincides with the full rationality (as there is no room for incorrect beliefs), and the weak Strotz model reduces to the Strotz model under full rationality.

[2] However, neither work is nested by the other. The present paper is restrictive by focusing on simple sequential games where no player moves more than once in a history, whereas Ray and Zhou [10] limits their attention into an extensive game with a unique equilibrium.

choice theoretic study. In Section 3, in turn, I define the individually rational choice in the collective choice environment and give some examples that illustrate such choice behavior with a rational player at the initial node. The section also provides a set of axioms that forms a necessary and sufficient condition for the individual rationality. Section 4 makes an adjustment to the previous result for obtaining a characterization of the individual rationality for an arbitrary player not necessarily at the initial node. Then, it is shown that if every player in a sequential game is individually rational, the observed choice of paths coincide with a set of subgame perfect equilibrium paths. Section 5 closes the paper with a few remarks. All proofs are left in the appendix.

2 Preliminaries

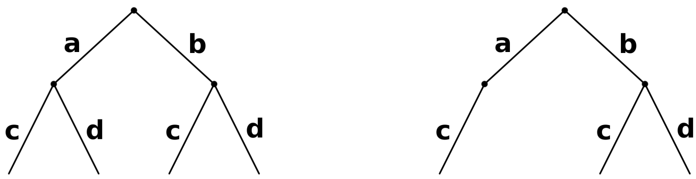
2.1 Choice and preference relations

For an arbitrary collection $\mathcal{X}$ of nonempty sets, *a choice correspondence* on $\mathcal{X}$ is a map $C : \mathcal{X} \rightrightarrows \bigcup \mathcal{X}$ such that $\varnothing \neq C(S) \subseteq S$ for every $S \in \mathcal{X}$. A choice correspondence is used to describe the decision maker's choice behavior in the sense that, when a set of available alternatives is given by $S \in \mathcal{X}$, the decision maker is willing to choose any alternative in $C(S)$ from S. A choice correspondence is often particularly interested when it is in a certain manner associated with a preference relation of the decision maker. Throughout the paper, *a preference relation* on Z refers to a complete and transitive binary relation on Z, where Z is an arbitrary nonempty set. For example, when $\mathcal{X}$ is the collection of all nonempty subsets of Z, a choice correspondence C on $\mathcal{X}$ is said to be *rationalizable* if there is a preference relation $\succcurlyeq$ on Z such that $C(S) = \{x \in S : x \succcurlyeq y \text{ for all } y \in S\}$ for every $S \in \mathcal{X}$.

2.2 Games

In this paper, we assume observability on the collective choice of paths from a variety of sequential games. A sequential game is a comprehensive description of a multi-player strategic environment that includes a set of players, a game tree, and an information structure. Note that players' preference relations are not included in the description of sequential games in order to study the revealed preference theory in a collective choice setup. The preference relations are instead revealed from observation of the collective choice behavior. Moreover, I place an assumption that we focus on sequential games of perfect information where players are aligned in the order $i = 1, 2, 3, \ldots$ and sequentially choose an action with knowledge of the actions chosen by the preceding players. Under these assumptions, we can identify sequential games with game trees which describe available paths that the players may choose.

With the preceding paragraphs in mind, let X be a set of infinite sequences. Every member $p = (p_1, p_2, \ldots)$ of X is interpreted as *a path* of actions taken by the players, and p_i is *an action* chosen by player i for every $i \in \mathbb{N}$. For notational convenience, I will write p^t to denote the first t terms of a path p, i.e. $(p_1, \ldots, p_t)$, for any $p \in X$ and $t \in \mathbb{N}$. The class X is used to represent the set of *conceivably all* paths that the players may follow without any feasibility constraint. However, the players may face some restrictions on the feasible paths, and this restriction is represented by a subset G of X that collects available paths for choice. Now, *a sequential game* (*a game* for short) is defined as a nonempty subset G of X with a certain closedness property.



(a) (b)

Figure 1: Sequential games

Definition. A sequential game is a nonempty subset G of X such that $p \in G$ whenever, for every $t \in \mathbb{N}$, there is a $q \in G$ with $q^t = p^t$.

Example 1. Let A_i be a nonempty set of actions chosen by player i for each $i \in \mathbb{N}$, and let $X = A_1 \times A_2 \times \cdots$. This is the case where, without any feasibility constraint, the set of actions that a player can choose does not depend on the actions chosen by the preceding players. For example, Figure 1(a) illustrates the set X when $A_1 = \{a, b\}$, $A_2 = \{c, d\}$, and A_i is singleton for all $i \geq 3$. Every sequence of branches from the initial node (top) to the terminal node (bottom) in the figure corresponds to a path in X. The illustration of trivial choices of actions by players $i \geq 3$ is omitted. Figures 1(a) and 1(b) are two examples of sequential games. In the latter game, an action d is not available for player 2 after player 1 chooses an action a.

Remark. As it is implied in Example 1, the present framework does not lose generality with respect to the number of players. An environment that involves only finitely many players can be formulated by augmenting players each of whom has a single artificial action interpreted as "no action". Formally, if the collection of conceivably all paths in the environment is given by a set Y of sequences of finite length $T \in \mathbb{N}$, we can proceed the analysis by setting $X = \{p : p^T \in Y \text{ and } p_t = \text{"no action" for all } t > T\}$.

Example 2 (Tree cutting problem)**.** Let $X = \{\mathbf{0}, \mathbf{e}_1, \mathbf{e}_2, \ldots\}$, where $\mathbf{0}$ denotes an infinite sequence of zeros, and $\mathbf{e}_i$ is the ith unit vector in ℓ^∞ for all $i \in \mathbb{N}$. (See Figure 2.) In association with a tree cutting problem, we interpret $\mathbf{0}$ as a path where the tree is never cut and $\mathbf{e}_i$ as where the tree is cut by the ith player. Now, let G be a subset of X such that $\{\mathbf{e}_1, \mathbf{e}_2, \ldots\} \subseteq G$, and suppose that G is a sequential game. In this game, every player can choose to cut or not to cut the tree provided that it has not been cut by the preceding players. Thus, in particular, a path in which all the players choose not to cut the tree should be feasible in the game. Indeed, since $\mathbf{e}_{t+1}^t = \mathbf{0}^t$ for all $t \in \mathbb{N}$, the closedness property of a sequential game implies that $\mathbf{0} \in G$.

Let $\mathcal{G}$ be the collection of all sequential games. Once the players face a sequential game G in $\mathcal{G}$, they collectively choose a path of actions in the following manner. First, player 1 chooses her action a_1. The action a_1 must be feasible in the sense that there is a path $p \in G$ with $p_1 = a_1$. Subsequently, player 2 chooses his action a_2, knowing that player 1 chose the action a_1. His action has to be feasible after a history a_1 in the sense that there is a path $p \in G$ with $p^2 = (a_1, a_2)$. Inductively, for every $t \in \mathbb{N}$, provided that the players up to t have chosen their actions $(a_1, \ldots, a_t)$, player $t + 1$ chooses her feasible action a_{t+1} such that $p^{t+1} = (a_1, \ldots, a_t, a_{t+1})$ for some $p \in G$. As a result of this procedure, a path $q = (a_1, a_2, \ldots)$ of actions is chosen by the players from the game G, where $q \in G$ by the closedness property. Each player is responsible for the choice of

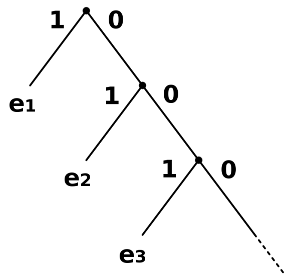


Figure 2: A tree cutting problem

her action, but she has no direct control for the other players' actions. Observability is assumed for the collective choice behavior from each sequential game in $\mathcal{G}$. In other words, the paper treats a choice correspondence C on $\mathcal{G}$ as observable data.

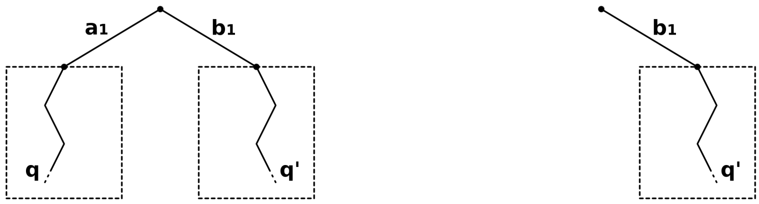
3 Individual rationality

This paper poses the following question: When we observe and only observe a choice correspondence C on $\mathcal{X}$ as a summary of collective choice behavior from sequential games in $\mathcal{X}$, how can we test whether an individual player involved in this environment is rational independently of the other players' rationality? In case where the players' preference relations are observable, and where we assume that all players are rational, various solution concepts in the game theory provide reasonable predictions for the collective choice (such as a subgame perfect equilibrium). However, the preference relations are not directly observable from choice data, and often there are some players, if not all, in reality who fail to comply the full rationality assumption. Therefore, it is important that we can test the rationality of an individual player involved in the games, and that we can reveal a preference relation from observable data, regardless of whether the other players are rational or not.

Approaching the problem, let us consider a player who chooses an action first (i.e. player 1), and assume that she rationally makes decision under a preference relation $\succcurlyeq$ on the set X. Note that the preference relation of the rational player may depend not only on her own action but on a complete path of actions chosen by all players. With the individual rationality, it is still difficult to predict the choice of paths in an arbitrary sequential game in $\mathcal{X}$ since nothing is assumed for behavior of players $i \geq 2$. However, the rationality of player 1 imposes a certain structure on the choice correspondence C as a result of her rational decision making.

For example, consider a game $G \in \mathcal{G}$ such that $|\{p_1 : p \in G\}| = 2$. (The game is depicted by Figure 3(a) where continuation games after player 1's actions are abstracted.) In this game, player 1 may make a choice from two distinct actions denoted by a_1 and b_1 at the initial node, though she has no direct control for the subsequent actions. Without loss of generality, suppose that we observe a choice of a path q from the game G, where player 1 chooses an action $q_1 = a_1$. Then, by the rationality assumption, player 1 knows that she does not better off by choosing an action b_1 instead. To be more precise, she should believe that, in case she would have chosen b_1, the subsequent players follow some path q' with $q'_1 = b_1$ which is no superior to q for player 1, namely, $q \succcurlyeq q'$.

Moreover, player 1, as a rational decision maker, must correctly form a belief about



(a) (b)

Figure 3: A sequential game with two actions at the initial node

actions taken by the others (regardless of whether they are chosen rationally), and the path q' is therefore one that the other players would indeed follow after a history where player 1 chose an action b_1 in the game G. But this is *observable*. Let $G_{b_1} = \{p \in G : p_1 = b_1\}$ be the same game as G except that player 1 is forced to play an action b_1. (See Figure 3(b).) As G_{b_1} is also a game, if player 1 correctly forms a belief about the other players' actions, we must observe that $q' \in C(G_{b_1})$. Therefore, although we may not pin down the collective choice for a given game from the individual rationality of player 1, her rationality entails a certain relation across observations from games.

I define a model of collective choice behavior where player 1 is rational as follows.

Definition. A choice correspondence C on $\mathcal{G}$ is said to be *individually rational* (at the initial node) if there exists a preference relation $\succcurlyeq$ on X such that, for all $G \in \mathcal{G}$, the following two statements are equivalent for an arbitrary $q \in G$.

(a) $q \in C(G)$.

(b) $q \in C(G_{q_1})$, and for any $b_1 \neq q_1$ in $\{p_1 : p \in G\}$, there is a $q' \in C(G_{b_1})$ with $q \succcurlyeq q'$,

where, for any action $a \in \{p_1 : p \in G\}$, G_a denotes the game $\{p \in G : p_1 = a\}$.

Example 3. Let A be a nonempty set, and X be the set of paths of form $p = (a, \emptyset, \emptyset, \ldots)$ for some $a \in A$, where the $\emptyset$ denotes an artificial action representing "no action". (See remark below Example 1.) For an arbitrary sequential game $G \subseteq X$, only player 1, if any, has nontrivial choice of actions, and therefore the game is essentially an individual choice problem. In this case, a choice correspondence C on $\mathcal{G}$ is individually rational if and only if there exists a preference relation $\succcurlyeq$ on A such that, for any game $G \in \mathcal{G}$, $C(G)$ is the set of paths $(a, \emptyset, \emptyset, \ldots)$ where an action a is such that $a \succcurlyeq p_1$ for any $p \in G$. Therefore, the individual rationality in the individual choice setup is a special instance of the individual rationality in the present collective framework.

Example 2 (continued)**.** Suppose that, in the tree cutting problem, every player prefers that a corresponding next player cuts the tree rather than doing so by herself, but she would prefer to cut the tree if it is otherwise cut by another player. So, denoting player t's preference relation by $\succcurlyeq_t$, $\mathbf{e}_{t+1} \succ_t \mathbf{e}_t \succ_t \mathbf{e}_s$ for all s distinct from t and $t+1$, and $\mathbf{e}_t \succ_t \mathbf{0}$. Suppose, moreover, that all except the initial player are myopic in the sense that they do not cut the tree as long as the next player has an option to cut, hoping that the next player does cut the tree without checking the consistency of such a belief. (For instance, the tree will never be cut in a game $G = \{\mathbf{0}, \mathbf{e}_2, \mathbf{e}_3, \ldots\}$.) In this environment, a choice correspondence that is individually rational at the initial node chooses a path by

$$C(G) = \begin{cases} \mathbf{e}_1 & \text{if } \mathbf{e}_1, \mathbf{e}_3 \in G, \\ \mathbf{e}_{t^*} & \text{otherwise} \end{cases} \qquad \text{for any } G \in \mathcal{G},$$

where t^* is the first t such that $\mathbf{e}_t \in G$ and $\mathbf{e}_{t+1} \notin G$. In particular, if $\mathbf{e}_3 \in G$, player 1 cuts the tree whenever possible, rationally expecting that player 2 will not cut the tree. In this environment, players $t \geq 2$ are all boundedly rational, but player 1 is nevertheless rational with a correct belief of actions taken by the other players.

Example 4. A choice correspondence can be individually rational even when players $t \geq 2$ do not have preference relations. For example, consider an environment where, without any feasibility restriction, player 1 can choose either to quit (0) or to continue (1), and player 2 can choose from three alternatives a, b, and c in case player 1 chose to continue the game. No player $t \geq 3$ has nontrivial choice of actions, and the game also ends immediately in case player 1 choose to quit. (See remark below Example 1.) Abusing notation, I identify a path $(0, \varnothing, \varnothing, \varnothing, \ldots)$ with 0 and $(1, x, \varnothing, \varnothing, \ldots)$ with x for $x = a, b, c$, and let X be the set of four paths $\{0, a, b, c\}$. Suppose that player 2's choice of actions depends on a set of available actions as follows.

$$C(\{a, b\}) = \{a\}, \quad C(\{a, c\}) = \{a\}, \quad C(\{b, c\}) = \{b\}, \quad C(\{a, b, c\}) = \{b\}.$$

This is what is called the choice pattern of a pairwise dominated alternative, and we can easily verify that there is no preference relation for player 2 that rationalizes his choice behavior. (This choice pattern is particularly interested in the literature in behavioral economics and marketing. See Manzini and Mariotti [8] and Masatlioglu et al. [9] for example.) Now, assume that player 1 is endowed with a preference relation $\succcurlyeq$ such that $a \succ 0 \succ b \succ c$. Then, an individually rational choice correspondence chooses a path by

$$C(\{0, a, b\}) = C(\{0, a, c\}) = \{a\} \quad \text{and} \quad C(\{0, b, c\}) = C(\{0, a, b, c\}) = \{0\}.$$

In particular, player 1 choose to quit in a game $\{0, a, b, c\}$ even though a preferred path a is available, rationally expecting that player 2 would choose a path b in case the game is continued.

3.1 Revealed preference theory

The definition of an individually rational choice correspondence, however, involves an unobservable preference relation for player 1, and therefore it is unclear how we may test the player's rationality from observed data. Even if a choice correspondence is not individually rational under a given preference relation, this may be either due to misspecification of the preference relation or to genuine irrationality in the player's choice behavior. Only when we verify that a choice correspondence cannot be viewed as individually rational under *any* preference relation, we may conclude that the player at the initial node is not rational. But the number of possible preference relations is quite large even when the domain X is small, and it is not realistic to check if a choice correspondence is consistent with the individual rationality for every specification of a preference relation.[3] This argument leads to the need of the revealed preference theory which allows us to estimate a player's preference relation directly from observed data.

For the first step toward the revealed preference theory, we may quickly realize that a usual method in the individual choice theory is not applicable in the current collective choice framework. In the standard choice theory, a preference of one alternative over another is revealed by observation that the decision maker chooses the former when the latter is available, that is, $x \succcurlyeq y$ if there is a choice set S such that $x \in C(S)$ and $y \in S$.

[3]There are, for example, more than 541 many preference relations when $|X| = 5$. The number increases to more than seven millions when $|X| = 9$. See Bailey [2].

In the present framework, on the other hand, a choice correspondence is induced from decisions made by multiple players, and hence observed choice data may not reflects individual preferences over outcomes (cf. Examples 2 and 4).

A preference relation is, however, partially revealed in the current framework when the players face a game where essentially a single player is delegated for choice of a path. For example, consider a game that consists of only two paths q and q' such that $q_1 \neq q'_1$. In this game, player 1 may either choose an action q_1 or an action q'_1, but no subsequent player has nontrivial choice of actions. Therefore, choice observation from such games can be utilized for revealing a preference relation of an individual player. Given this revealed preference relation, the next axiom requires consistency of a choice correspondence with the individual rationality.

Axiom 1. Let $G \in \mathcal{G}$ and $q \in G$. Then, $q \in C(G)$ if and only if $q \in C(G_{q_1})$, and for any $b_1 \neq q_1$ in $\{p_1 : p \in G\}$, there is a $q' \in C(G_{b_1})$ with $q \in C(\{q, q'\})$.

Note that the axiom is identical to the definition of the individual rationality except that a condition $q \succcurlyeq q'$ is replaced by $q \in C(\{q, q'\})$. The axiom directly rephrases the individual rationality in terms of a revealed preference so that we can test an observed choice correspondence for rationalizability. However, it should be also remarked that a possibility of partially inferring a preference relation by examining choice behavior $C(\{q, q'\})$ with $q_1 \neq q'_1$ does not guarantee that player 1 is indeed endowed with a well defined preference relation. Axiom 1 only provides a necessary condition that must be met *if* player 1 in fact rational, and she is endowed with a well defined preference relation. An interesting problem of the revealed preference theory in the present framework therefore boils down to the next question.

Question. When is there a (well-defined) preference relation $\succcurlyeq$ on X that is consistent with arbitrary binary choice data $C(\{q, q'\})$ with $q_1 \neq q'_1$ in the sense below?

$$q \succcurlyeq q' \text{ if } q \in C(\{q, q'\}) \quad \text{and} \quad q \succ q' \text{ if } \{q\} = C(\{q, q'\}).$$

The main result of the paper provides the answer of this question in two axioms.

Axiom 2. Provided that $p, q, r \in X$ are such that $p_1 \neq q_1 \neq r_1 \neq p_1$, $p \in C(\{p, q\})$ and $q \in C(\{q, r\})$ imply $p \in C(\{p, r\})$.

Axiom 3. Provided that $p, q, r, s \in X$ are such that $p_1 = r_1 \neq q_1 = s_1$, $p \in C(\{p, q\})$, $q \in C(\{q, r\})$, and $r \in C(\{r, s\})$ imply $p \in C(\{p, s\})$.

Theorem 1. *A choice correspondence C on $\mathcal{G}$ is individually rational if and only if it satisfies Axioms 1 through 3.*

Therefore, the individual rationality is falsifiable by testing the three axioms. The rationality concept proposed here is a direct extension of the individual rationality in an individual choice environment, and it reduces to the standard preference maximization model when there is essentially a single nontrivial player (see Example 3). However, more importantly, the theorem also assures testability of the individual rationality in the current collective choice environment, and it allows us to test whether an individual player is rational or not, independently of the other players' rationality. This implies that the axioms are weaker than testing certain collective solution concepts such as the subgame perfect equilibrium, and they can be used to classify players into those who are rational and those who are not. (See Section 4 for further discussion.) In the next section, we verify that when every player involved in the environment happens to be individually rational, their collective choice of paths is in fact viewed as *collectively rational*, meaning that it chooses a set of subgame perfect equilibrium paths. Therefore, the theorem also guarantees testability of the well-known collective solution concept as a special case where every individual player is rational.

4 Characterization of collective rationality

So far, we focus on the rationality of an individual player at the initial node (player 1), and Theorem 1 characterizes her individual rationality in the collective choice environment. In this section, we first observe that this result can be easily modified to provide a testable characterization for the rationality of an arbitrary player at an arbitrary decision node. Below, any finite sequence $\gamma = (\gamma_1, \ldots, \gamma_k)$ of actions for which there exists a path $p \in X$ with $p^k = \gamma$ is referred to as *a decision node* in X. Given a sequential game $G \in \mathcal{G}$, a decision node may or may not be achievable. In case a decision node γ is achievable in the game G, that is, there exists a path $p \in G$ with $p^k = \gamma$, we say that γ is a decision node in G. The sequence $\varnothing$ of zero length is also viewed as a decision node, and it is particularly called the initial node. The initial node is, of course, achievable in any sequential game.

Definition. Let C be a choice correspondence on $\mathcal{G}$, and $\gamma = (\gamma_1, \ldots, \gamma_k)$ be any decision node in X. Define tuple $(X_\gamma, \mathcal{G}_\gamma, C_\gamma)$ by

(a) $X_\gamma = \{p : (\gamma, p) \in X\}$,

(b) $\mathcal{G}_\gamma$ is the set of all sequential games in X_γ,

(c) $C_\gamma(G) = \{p \in G : (\gamma, p) \in C(\gamma, G)$ for all $G \in \mathcal{G}_\gamma\}$ for all $G \in \mathcal{G}_\gamma$,

where $(\gamma, p) = (\gamma_1, \ldots, \gamma_k, p_1, p_2, \ldots)$ and $(\gamma, G) = \{(\gamma, p) : p \in G\}$. Then, C is said to be *individually rational* at the decision node γ if C_γ is individually rational at the initial node. In turn, we say that C is *collectively rational* if C is individually rational at every decision node in X.

In the definition, the ternary tuple $(X_\gamma, \mathcal{G}_\gamma, C_\gamma)$ forms a truncated data that records the collective choice behavior $(X, \mathcal{G}, C)$ after the decision node γ. The collection X_γ is the set of all infinite paths of actions that are feasible in X after the decision node γ, and $\mathcal{G}_\gamma$ is the collection of all sequential games in X_γ. Then, C_γ is a choice correspondence on $\mathcal{G}_\gamma$ such that, for any sequential game G in $\mathcal{G}_\gamma$, C_γ chooses a path p from G if and only if the original choice data C chooses a path (γ, p) from (γ, G).

Remark. Regarding the truncated choice data, two remarks are worth noted. First, apart from the interpretation of the data, a mathematical structure of the data $(X_\gamma, \mathcal{G}_\gamma, C_\gamma)$ fits in the model developed in Section 3. Therefore, not only the individual rationality of C_γ is meaningful, Theorem 1 also applies to provide the axiomatic characterization of the individual rationality for C_γ. Second, upon defining $(X_\gamma, \mathcal{G}_\gamma, C_\gamma)$, no information other than the original choice data $(X, \mathcal{G}, C)$ is required. Therefore, in order to test the individual rationality of C_γ at the initial node, or equivalently, the individual rationality of C at the decision node γ, we need to make no additional observability assumption.

The main result of this section proves that, when the collective choice behavior C is collectively rational (meaning that C is individually rational at every decision node), C in fact chooses subgame perfect equilibrium paths from any sequential game $G \in \mathcal{G}$.

This result on the one hand gives fine justification to the individual rationality concept defined in this paper. On the other hand, the result at the same time provides a testable axiomatic characterization for the collective solution concept widely used in the game theory. In the theorem below, the definition of subgame perfect equilibrium paths are postponed to Appendix as this notion is quite well known in the literature.

Theorem 2. *If a choice correspondence C on $\mathcal{G}$ is collectively rational, then there is a sequence of preference relations $(\succcurlyeq_i)$ on X such that, for every sequential game $G \in \mathcal{G}$, $C(G)$ consists of subgame perfect equilibrium paths in G under $(\succcurlyeq_i)$.*

Ray and Zhou [10] study an axiomatic characterization of a single-valued choice function that chooses the unique subgame perfect equilibrium under a sequence of strict preference relations. In allowing the possibility of indifference, therefore, Theorem 2 generalizes their result by providing the characterization of collectively rational choice correspondences. On the other hand, Ray and Zhou consider general extensive games with perfect information where the same player may choose actions more than once through a history, which is excluded by the choice environment studied in this paper. So, there is formally no nested relation between these two results.[4]

5 Conclusion

The standard game theory relies on the premise of rational players and availability of payoff functions known to the outside observers. Provided with these assumptions, the theory offers a strong predictive power in the form of various solution concepts such as Nash equilibrium in simultaneous games or subgame perfect equilibrium in extensive games. However, empirical and experimental evidence often suggests that some players in the game, if not all, behave differently from the theoretical predictions, and in such a case we may find it difficult to support the rationality hypothesis as a convincing assumption imposed on *every* existing player. Moreover, what is observable in practise is typically restricted to choice data of actions, and preference relations for the players are not directly observable. It is, therefore, of importance for empirical contents of the theory that the rationality of the players is testable and that their preference relations can be revealed from observable choice data.

In this paper, I studied testable axiomatization of the individual rationality for the players involved in the collective choice environment. Notably, the individual rationality is in this paper so defined that it does not rely on the rationality of the other players, and therefore the paper provides a method that enables us to identify players who can be viewed as rational independently of the behavioral patterns of the others. In addition, I also show that, in a special case where every player in the choice environment happens to be individually rational, the players collectively choose action profiles that coincide with the prediction of a solution concept in the game theory. So, the axioms characterizing the individual rationality turn out to be useful for testing the collective rationality of the players.

In closing, I note that testable characterization of individual rationality in general choice setup has not been given in the literature to my knowledge. The paper focuses on

[4]The work of Ray and Zhou focuses on extensive games that always end after finitely many actions. In the terminology of this paper, this condition can be stated by a property of X that, for all $p \in X$, there exists $t \geq 0$ such that $q^t = p^t$ implies $q = p$ for any $q \in X$. We can show that, if X meets this property, Theorem 2 can be strengthened to the following equivalent characterization result: *A choice correspondence C on $\mathcal{G}$ is collectively rational if and only if there exists a sequence of preference relations $(\succcurlyeq_i)$ on X such that, for every sequential game $G \in \mathcal{G}$, $C(G)$ is the set of all subgame perfect equilibrium paths in G under $(\succcurlyeq_i)$.*

the study of collective choice behavior from sequential games of perfect information, in which players choose their actions in a prespecified order with the information of the actions chosen by the preceding players. This framework is of interest by including a multi-period consumption problem under a time-inconsistent preference relation (Example 2) and certain menu choice frameworks (Example 4), but it also excludes other game forms of interest. Therefore, finding an axiomatic characterization of individual rationality in other game forms (which, preferably, reduces to a known equilibrium concept in the collective choice theory if all players are rational) is an open problem.

Appendix

Proof of Theorem 1. The "only if" part of the claim follows easily from the representation, and the proof is omitted. For the "if" part, let C be a choice correspondence on $\mathcal{G}$ that satisfies Axioms 1 through 3. Define a binary relation $\succcurlyeq$ on X by $p \succcurlyeq q$ if and only if $p_1 \neq q_1$ and $p \in C(\{p, q\})$. To show that $\succcurlyeq$ is acyclic, suppose, by contradiction, that $p^1 \succcurlyeq p^2 \succcurlyeq \cdots \succcurlyeq p^k \succ p^1$ for some finite sequence (p^l) in X. First, assume that $k \geq 4$. If $p_1^1 \neq p_1^3$, then $p^1 \succcurlyeq p^3$ by Axiom 2. Also, if $p_1^2 \neq p_1^4$, then $p^2 \succcurlyeq p^4$ by the same axiom. On the other hand, if $p_1^1 = p_1^3$ and $p_1^2 = p_1^4$, we have $p^1 \succcurlyeq p^4$ by Axiom 3. Since these three cases exhaust all the possibilities, it turns out that, whenever $k \geq 4$, the sequence (p^l) is shrinkable while it remains to be a $\succcurlyeq$-cycle. So, we can assume that $k = 3$ without loss of generality. But then $p^1 \succcurlyeq p^2 \succcurlyeq p^3 \succ p^1$, which gives a contradiction to Axiom 2. It follows that $\succcurlyeq$ is acyclic. Now, by Szpilrajn's theorem [15], there exists a complete and transitive binary relation $\succcurlyeq^*$ on X that extends $\succcurlyeq$. The individual rationality of the choice correspondence C follows from Axiom 1 at once under this preference relation $\succcurlyeq^*$. $\square$

Subgame perfect equilibria. Let $(\succcurlyeq_i)$ be an infinite sequence of complete and transitive binary relations on X with an interpretation that each $\succcurlyeq_i$ is a preference relation for player i. Let G be any sequential game in $\mathcal{G}$, and denote by Γ_G the set of decision nodes in G. A (pure) strategy profile in the sequential game G describes an action chosen by a corresponding player at every decision node in the game G. Noting that the choice of an action by the player leads the game to another decision node in G, *a strategy profile* is formally defined as a self-map σ on Γ_G such that $\sigma(\gamma) = (\gamma_1, \ldots, \gamma_k, a)$ for every decision node $\gamma \in D_G$. Now, let σ^0 be the identity function, and $\sigma^{t+1} = \sigma^t \circ \sigma$ for all $t \geq 0$. Given any $\gamma \in D_G$ of length k, a strategy profile σ in G induces a path p such that $p^{k+t} = \sigma^t(\gamma)$ for each $t \geq 0$. We shall denote by $\phi(\sigma, \gamma)$ this path p induced by σ from the node γ. Note that $\phi(\sigma, \gamma) \in G$ by the closedness property of sequential games.

Definition. A strategy profile σ in a sequential game $G \in \mathcal{G}$ is called *a subgame perfect equilibrium* under the sequence $(\succcurlyeq_i)$ of preference relations if, for all $k \geq 0$, all $\gamma \in \Gamma_G$ of length k, and all a with $(\gamma, a) \in \Gamma_G$, $\phi(\sigma, \gamma) \succcurlyeq_{k+1} \phi(\sigma, (\gamma, a))$. In turn, a path $p \in G$ is called *a subgame perfect equilibrium path* if there exists a subgame perfect equilibrium strategy profile σ in G such that $p = \phi(\sigma, \emptyset)$.

Proof of Theorem 2. We first define, for each $i \geq 1$, player i's preference relation $\succcurlyeq_i$. For this, fix any $i \geq 1$. Since C is collectively rational by hypothesis, for every decision node γ of length $i - 1$, there is a preference relation $\succcurlyeq_\gamma$ on X_γ under which C_γ is individually rational. Define a transitive relation $\trianglerighteq_\gamma$ on X by $p \trianglerighteq_\gamma q$ iff $p^{i-1} = q^{i-1} = \gamma$ and $(p_i, p_{i+1}, \ldots) \succcurlyeq_\gamma (q_i, q_{i+1}, \ldots)$, and let $\trianglerighteq_i$ be the union of $\trianglerighteq_\gamma$ across all decision nodes γ of length $i - 1$. It is easy to see that $\trianglerighteq_i$ is a reflexive. Furthermore, if $p \trianglerighteq_i q \trianglerighteq_i r$, then $p \trianglerighteq_\gamma q \trianglerighteq_\gamma r$ for a unique decision node γ of length $i-1$, and thus we have $p \trianglerighteq_\gamma r$ and $p \trianglerighteq_i r$, proving transitivity of $\trianglerighteq_i$. By Szpilrajn's theorem [15], any reflexive and transitive binary relation on X admits a completion.[5] So, let $\succcurlyeq_i$ be a completion of $\trianglerighteq_i$. As $i \geq 1$ is arbitrary, we have constructed a sequence $(\succcurlyeq_i)$ of preference relations on X.

Next, we show that, for all $G \in \mathcal{G}$, every path in $C(G)$ is a subgame perfect equilibrium path under $(\succcurlyeq_i)$. Fix any sequential game $G \in \mathcal{G}$, and let $p \in C(G)$. For each $t \geq 0$, denote the set of all decision nodes of length t by Γ^t_G. Below, for all $t \geq 0$, we define a function f_t that maps each decision node γ in Γ^t_G to a path in $C_\gamma(\{q : (\gamma, q) \in G\})$. (Interpretation: $f_t(\gamma)$ is a path that the rational players would follow if the game reaches the decision node γ.) First, Γ^0_G is a singleton set of the initial node $\varnothing$. Let $f_0(\varnothing) = p$. Then, by induction, suppose that f_t is defined for some $t \geq 0$. To define f_{t+1}, take any $\gamma \in \Gamma^{t+1}_G$. Then, $\gamma^t = (\gamma_1, \ldots, \gamma_t)$ is a decision node in Γ^t_G, and $f_t(\gamma^t) =: q^*$ is a path in $C_{\gamma^t}(\{q : (\gamma^t, q) \in G\})$. If $q^*_1 = \gamma_{t+1}$, then let $f_{t+1}(\gamma) = (q^*_2, q^*_3, \ldots)$. Otherwise, because $q^* \in C_{\gamma^t}(\{q : (\gamma^t, q) \in G\})$, the individual rationality of C_{γ^t} guarantees the existence of a path q' in $C_{\gamma^t}(\{q : (\gamma^t, q) \in G\}_{\gamma_{t+1}})$ such that $(\gamma^t, q^*) \succcurlyeq_{t+1} (\gamma^t, q')$. In this case, define $f_{t+1}(\gamma) = (q'_2, q'_3, \ldots)$, and note that $f_{t+1}(\gamma) \in C_{\gamma^{t+1}}(\{q : (\gamma^{t+1}, q) \in G\})$ by construction of $C_{\gamma^{t+1}}$. As $\gamma \in \Gamma^{t+1}_G$ is chosen arbitrarily, a function f_{t+1} is fully defined on Γ^{t+1}_G, and we now have a sequence of functions (f_t) by induction. Noting that (Γ^t_G) is a partition of Γ_G, a strategy profile σ is defined by $\sigma(\gamma) = (\gamma, f_t(\gamma)_1)$ for every $t \geq 0$ and $\gamma \in \Gamma^t_G$. Then, $\phi(\sigma, \gamma) = (\gamma, f_t(\gamma))$ for every $t \geq 0$ and $\gamma \in \Gamma^t_G$, and moreover it follows that

$$\phi(\sigma, \gamma) = (\gamma, f_t(\gamma)) \succcurlyeq_{t+1} (\gamma, f_{t+1}(\gamma, a)) = \phi(\sigma, (\gamma, a))$$

for any $t \geq 0$, $\gamma \in \Gamma^t_G$, and a with $(\gamma, a) \in \Gamma^{t+1}_G$. So, σ is a subgame perfect equilibrium in G, and $p = f_0(\varnothing) = \phi(\sigma, \varnothing)$ is a subgame perfect equilibrium path in G. As $G \in \mathcal{G}$ and $p \in C(G)$ is arbitrary, the proof is complete. □

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